

1 次の関数を微分せよ。

(1) $y = \sin(3x+2)$

$$y' = 3 \cos(3x+2)$$

(2) $y = \frac{1}{2x+3}$

$$y' = -\frac{2}{(2x+3)^2}$$

(3) $y = \sqrt[3]{x^2+4}$

$$y = (x^2+4)^{\frac{1}{3}}$$

$$\begin{aligned} \text{∴} y' &= \frac{1}{3}(x^2+4)^{-\frac{2}{3}} \cdot 2x \\ &= \frac{2x}{3\sqrt[3]{(x^2+4)^2}} \end{aligned}$$

(4) $y = x \log 3x$

$$\begin{aligned} y' &= 1 \cdot \log 3x + x \cdot 3 \cdot \frac{1}{3x} \\ &= \log 3x + 1 \end{aligned}$$

(5) $y = xe^{-x^2}$

$$\begin{aligned} y' &= 1 \cdot e^{-x^2} + x \cdot (-2x) \cdot e^{-x^2} \\ &= e^{-x^2}(1-2x^2) \end{aligned}$$

(6) $y = \log |\tan x|$

$$\begin{aligned} y' &= \frac{1}{\tan x} \cdot (\tan x)' \\ &= \frac{1}{\tan x} \cdot \frac{1}{\cos^2 x} \\ &= \frac{1}{\sin x \cdot \cos x} \end{aligned}$$

(7) $y = x^x \ (x > 0)$

$$\log y = x \cdot \log x$$

両辺微分し

$$\frac{y'}{y} = \log x + 1$$

$$y' = y \cdot (\log x + 1)$$

$$y' = x^x \cdot (\log x + 1)$$

(8) $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$$\begin{aligned} y' &= \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2} \\ &= \frac{e^{2x} + 2 + e^{-2x} - (e^{2x} - 2 + e^{-2x})}{(e^x + e^{-x})^2} \\ &= \frac{4}{(e^x + e^{-x})^2} \end{aligned}$$

2 次の関数を微分せよ。ただし、(8)のaは定数とする。

(1) $y = (2x+1)\sqrt{x^2-1}$

$$\begin{aligned} y' &= 2\sqrt{x^2-1} + (2x+1) \cdot \frac{1}{2}(x^2-1)^{-\frac{1}{2}} \cdot 2x \\ &= 2\sqrt{x^2-1} + \frac{(2x+1) \cdot x}{\sqrt{x^2-1}} \\ &= \frac{2(x^2-1) + 2x^2 + x}{\sqrt{x^2-1}} \\ &= \frac{4x^2 + x - 2}{\sqrt{x^2-1}} \end{aligned}$$

(2) $y = \frac{\sin x}{1 + \cos x}$

$$\begin{aligned} y' &= \frac{\cos x \cdot (1 + \cos x) - \sin x \cdot (-\sin x)}{(1 + \cos x)^2} \\ &= \frac{1 + \cos x}{(1 + \cos x)^2} \\ &= \frac{1}{1 + \cos x} \end{aligned}$$

(3) $y = \tan^2 3x$

$$\begin{aligned} y' &= 2 \tan 3x \cdot (\tan 3x)' \\ &= 2 \tan 3x \cdot \frac{1}{\cos^2 3x} \cdot 3 \\ &= \frac{6 \tan 3x}{\cos^2 3x} \end{aligned}$$

(4) $y = \log \left(\frac{1}{\cos x} \right)$

$$\begin{aligned} y' &= \frac{1}{\frac{1}{\cos x}} \cdot \left(\frac{1}{\cos x} \right)' \\ &= \cos x \cdot \frac{-(-\sin x)}{\cos^2 x} \\ &= \tan x \end{aligned}$$

(5) $y = \log_{10}(x^2+1)$

$$y = \frac{\log(x^2+1)}{\log 10}$$

$$\begin{aligned} \text{∴} y' &= \frac{1}{\log 10} \cdot \frac{1}{x^2+1} \cdot 2x \\ &= \frac{2x}{(x^2+1) \cdot \log 10} \end{aligned}$$

(6) $y = e^{2x} \cos 3x$

$$\begin{aligned} y' &= 2 \cdot e^{2x} \cos 3x + e^{2x} \cdot (-\sin 3x) \cdot 3 \\ &= e^{2x}(2 \cos 3x - 3 \sin 3x) \end{aligned}$$

(7) $y = (\sqrt{x})^x \ (x > 0)$

$$\log y = \frac{1}{2} x \cdot \log x$$

両辺を微分し

$$\frac{y'}{y} = \frac{1}{2} (\log x + x \cdot \frac{1}{x})$$

$$y' = \frac{1}{2} y \cdot (\log x + 1)$$

$$y' = \frac{1}{2} \sqrt{x}^x \cdot (\log x + 1)$$

(8) $y = \log(x + \sqrt{x^2+a^2})$

$$y' = \frac{1}{x + \sqrt{x^2+a^2}} \left(1 + \frac{1}{2}(x^2+a^2)^{-\frac{1}{2}} \cdot 2x \right)$$

$$\begin{aligned} &= \frac{1}{x + \sqrt{x^2+a^2}} \left(1 + \frac{x}{\sqrt{x^2+a^2}} \right) \\ &= \frac{1}{x + \sqrt{x^2+a^2}} \cdot \frac{\sqrt{x^2+a^2} + x}{\sqrt{x^2+a^2}} \end{aligned}$$

$$= \frac{1}{\sqrt{x^2+a^2}}$$